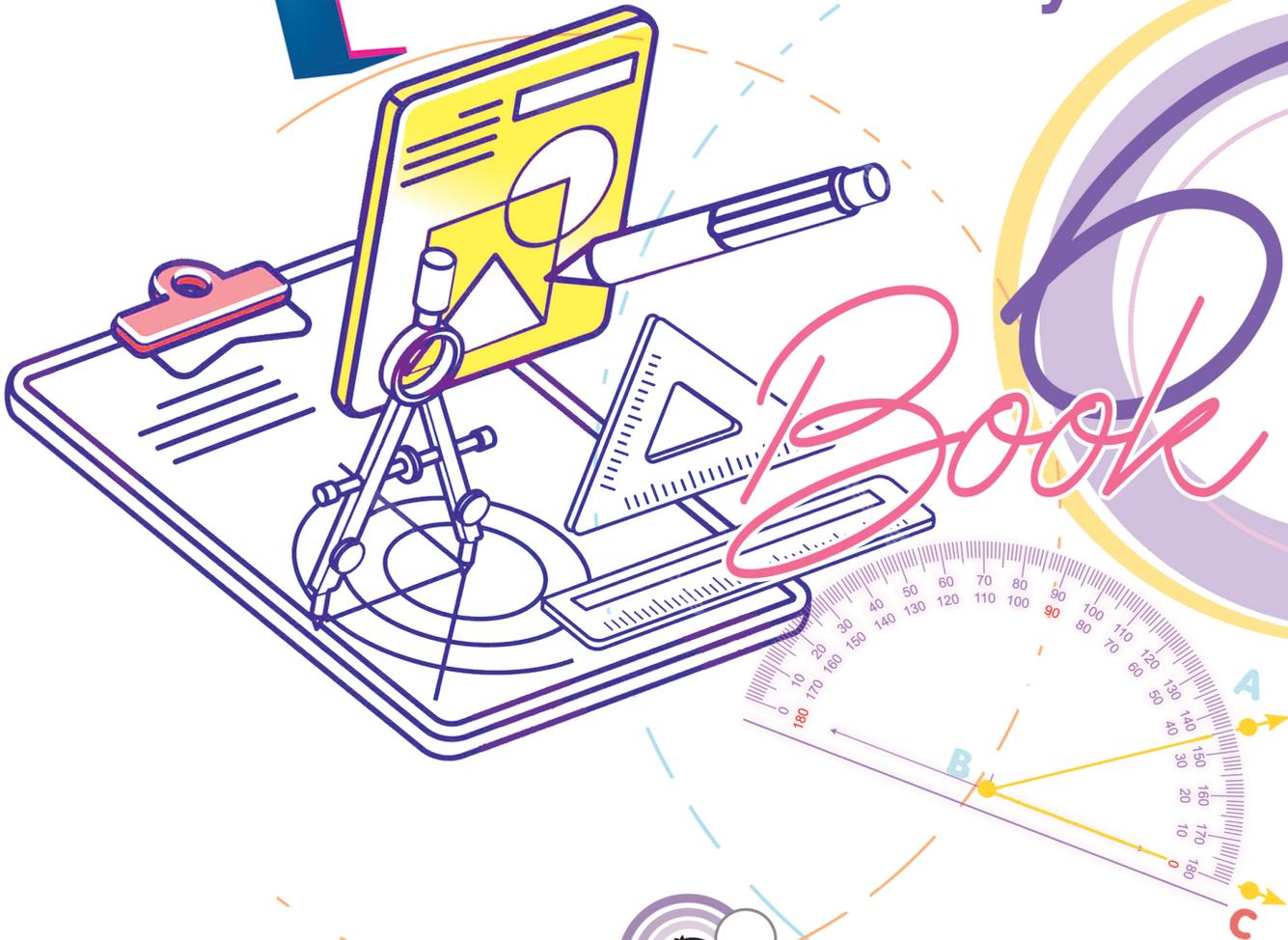


# Spark

# Math

## Answer Key



**Dar  
CHAD ALAJYAL**  
For Publishing & Distributing

Fourth Edition  
2024

# Spark of Math

## Answer Key Book 6

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## Unit 1

# Multiplication and Division

## Vocabulary



- multiplicand
- multiplier
- product
- dividend
- divisor
- quotient
- remainder (r)
- prime number
- composite number
- factor
- multiple
- factorization
- prime factorization
- greatest common factor (GCF)
- least common multiple (LCM)

## Objectives



**Upon completion of this unit, you will be able to:**

- Multiply numbers by 2- or 3-digit numbers.
- Divide numbers by 2-digit numbers.
- Solve multi-digit multiplication and division problems.
- Define prime and composite numbers.
- Identify all the factors of a given number.
- Identify a set of multiples of a given number.
- Determine the greatest common factor of two or more numbers.
- Determine the least common multiple of two or more numbers.



## (1-2) Division

Division is the inverse operation of multiplication. For instance, if multiplying 3 groups of 4 results in 12, then dividing 12 into 3 equal groups results in 4 in each group.

To find  $3784 \div 35$ :

3784  $\longleftarrow$  dividend  
 35  $\longleftarrow$  divisor  
 108  $\longleftarrow$  quotient  
 4  $\longleftarrow$  remainder (r)

$$\begin{array}{r}
 \times 108 \\
 35 \overline{) 3784} \\
 \underline{- 35} \phantom{0} \\
 28 \phantom{0} \\
 \underline{- 0} \phantom{0} \\
 284 \\
 \underline{- 280} \\
 r 4
 \end{array}$$

$\longrightarrow$  Divide 37 by 35. Write down 1 above the division bracket.  
 $\longrightarrow$  Multiply 1 by 35 to get 35.  
 $\longrightarrow$  Subtract 35 from 37. Bring down 8. Divide 28 by 35. Write down 0 above.  
 $\longrightarrow$  Multiply 0 by 35, then subtract 0 from 28, leaving 28.  
 $\longrightarrow$  Bring down 4. Divide 284 by 35. Write down 8 above.  
 $\longrightarrow$  Multiply 8 by 35.  
 $\longrightarrow$  Subtract 280 from 284. The remainder is 4.

To check the division result, multiply the quotient by the divisor and add the remainder. ( $108 \times 35 + 4 = 3784$ )

### 1. Divide, and then check your answers.

$$\begin{array}{r}
 \times 32 \\
 72 \overline{) 2315} \\
 \underline{- 216} \\
 155 \\
 \underline{- 144} \\
 11
 \end{array}$$

Check the answer.  
 $32 \times 72 + 11 =$   
 $2304 + 11 =$   
 $2315$

$$\begin{array}{r}
 \times 185 \\
 49 \overline{) 9083} \\
 \underline{- 49} \\
 418 \\
 \underline{- 392} \\
 263 \\
 \underline{- 245} \\
 18
 \end{array}$$

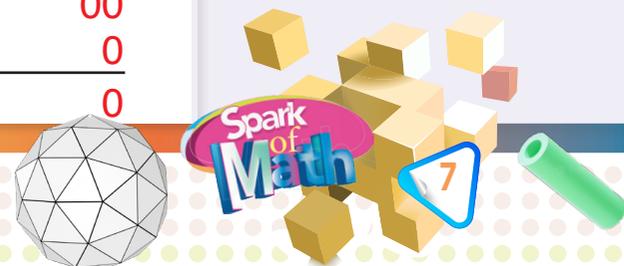
Check the answer.  
 $185 \times 49 + 18 =$   
 $9065 + 18 =$   
 $9083$

$$\begin{array}{r}
 \times 618 \\
 54 \overline{) 33372} \\
 \underline{- 324} \\
 97 \\
 \underline{- 54} \\
 432 \\
 \underline{- 432} \\
 0
 \end{array}$$

Check the answer.  
 $618 \times 54 + 0 =$   
 $33372$

$$\begin{array}{r}
 \times 690 \\
 18 \overline{) 12420} \\
 \underline{- 108} \\
 162 \\
 \underline{- 162} \\
 00 \\
 \underline{- 0} \\
 0
 \end{array}$$

Check the answer.  
 $690 \times 18 + 0 =$   
 $12420$



**2. Circle the correct answers.**

**A**  $585 \div 9 = 65$  (Yes / No)

**B**  $4200 \div 35 = 12$  (Yes / No)

**C**  $8579 \div 4 = 2144$  (Yes / No)

**D**  $930 \div 15 = 26$  (Yes / No)

**Your Work**

Students' own answers

**Fill in the blank to make the sentence true.  
Then discuss your answer with your classmates.**

$4,500 \div 7 > 500$

$500 \times 7 = 3500$

$4,500 > 3500$



## (1-3) Prime and Composite Numbers

Numbers are divided into two types according to their factors: prime and composite.

- A prime number has only two factors: 1 and itself.

**Example:**

5 is a prime number because the only factors it has are 1 and 5.

- A composite number has more than two factors.

**Example:**

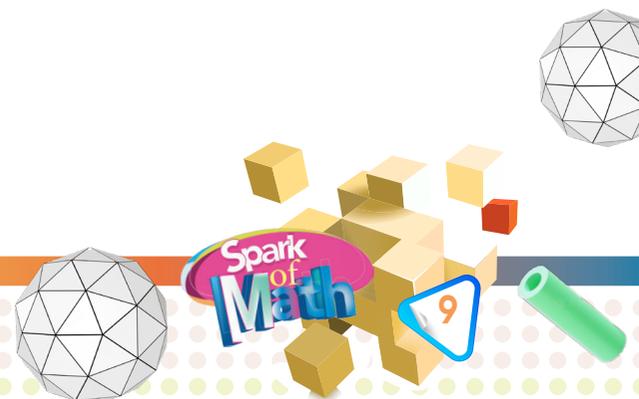
15 is a composite number because it has more than two factors: 15, 5, 3, and 1.

### 1. Circle all the prime numbers.

13 54 37 96 89 45 61  
29 2 10 69 39 53 47

### 2. Circle all the composite numbers.

73 92 79 7 21 80 17  
11 4 23 32 67 3 49



3. Circle the greatest prime number.

A 74

B 23

C 69

D 31

4. Circle the smallest composite number.

A 16

B 59

C 11

D 49

### Your Work

Circle the correct answers, then discuss them with your classmates.

A All even numbers are prime. (Yes / No)

All prime numbers are odd except for 2.

B All odd numbers are composite. (Yes / No)

Not all odd numbers are composite.

C Number 1 is neither prime nor composite. (Yes) / No

Because it has only one positive divisor (itself).



## (1-4) Factorization

### Factors

- Factors are numbers that you can multiply to get a product.

$$5 \times 8 = 40$$

- Think of two numbers that multiply together to give 40.

$$40 \times 1 = 40$$

$$20 \times 2 = 40$$

$$10 \times 4 = 40$$

$$8 \times 5 = 40$$

- The factors for 40 are 1, 2, 4, 5, 8, 10, 20, and 40.

### Multiples

- Multiples of a number are the results of multiplying that number by a whole number.

$$5 \times 8 = 40$$

- Multiples of a number follow a pattern of skip counting. The multiples of a number can go on infinitely.

$$5 \times 1 = 5$$

$$5 \times 2 = 10$$

$$5 \times 3 = 15$$

$$5 \times 4 = 20$$

- The multiples of 5 include 5, 10, 15, 20, and so on.

## Prime Factorization

Every number can be written as a product of prime numbers.

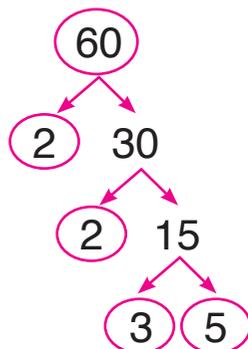
This product is called prime factorization. You can use a diagram called a factor tree or repeated division to find the prime factorization.

### Example:

$$60 = 2 \times 2 \times 3 \times 5$$

### Method 1

Factor Tree



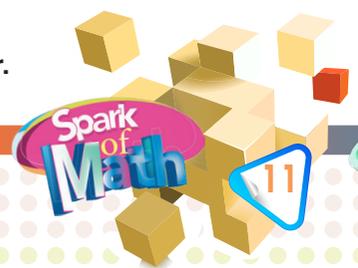
### Method 2

Repeated Division

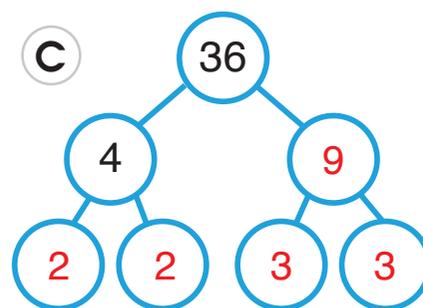
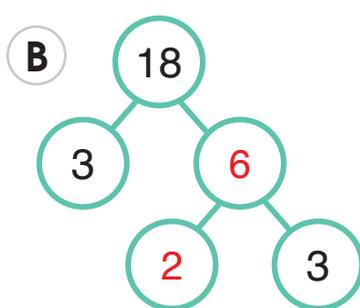
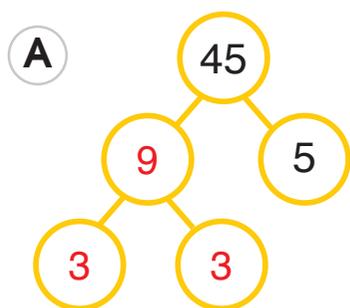
2	60	$60 \div 2 = 30$
2	30	$30 \div 2 = 15$
3	15	$15 \div 3 = 5$
5	5	$5 \div 5 = 1$
	1	

Note that when you multiply all the prime factors, you will get the original number.

$$2 \times 2 \times 3 \times 5 = 60$$



## 1. Complete the factor trees.



## 2. Find the prime factorization for each number using the repeated division method.

2	80	$(80 \div 2 = 40)$
2	40	$(40 \div 2 = 20)$
2	20	$(20 \div 2 = 10)$
2	10	$(10 \div 2 = 5)$
5	5	$(5 \div 5 = 1)$
	1	

2	64	$(64 \div 2 = 32)$
2	32	$(32 \div 2 = 16)$
2	16	$(16 \div 2 = 8)$
2	8	$(8 \div 2 = 4)$
2	4	$(4 \div 2 = 2)$
2	2	$(2 \div 2 = 1)$
	1	



3. Circle the numbers that are multiples of 5.

19    20    43    15    35    11    40

4. Circle the numbers that are multiples of 3.

6    23    18    20    30    24    13

5. Are multiples of 4 always even? Explain.

Yes, multiples of 4 are always even because any multiple of 4 can be divided by 2 without a remainder.

6. Are multiples of 3 always odd? Explain.

Multiples of 3 can be either odd or even, so they are not always odd.

### Your Work

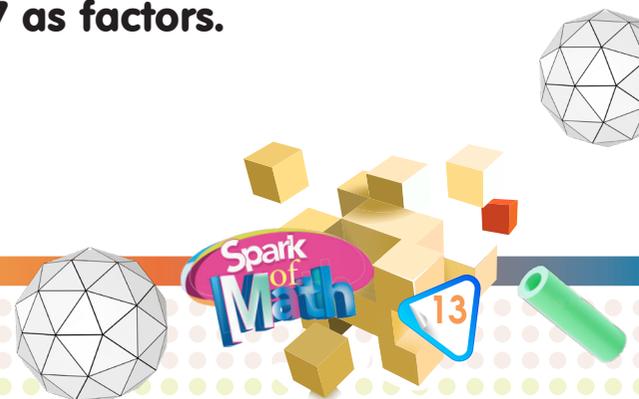
1. Find the prime factorization of 450.

$$450 = 5 \times 5 \times 2 \times 3 \times 3$$

5	450
5	90
2	18
3	9
3	3
	1

2. Write down numbers that have 5, 2, and 7 as factors.

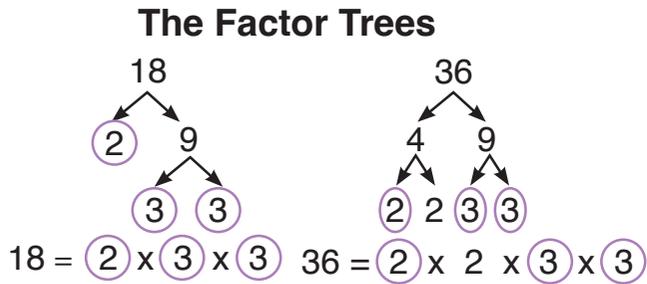
70, 140, 210, or 280.



## (1-5) The Greatest Common Factor (GCF)

The greatest common factor (GCF), also called the greatest common divisor (GCD), is the greatest factor shared between two or more numbers.

To find the GCF of 18 and 36, you can use the prime factorization method.



$$2 \times 3 \times 3 = 18$$

So, the GCF of 18 and 36 is 18.

**Repeated Division**

2	18
3	9
3	3
	1

2	36
2	18
3	9
3	3
	1

### 1. Determine the GCF of each set of numbers.

- A** 6 and 24

$$6 = 2 \times 3$$

$$24 = 2 \times 3 \times 2 \times 2$$

$$\text{GCF} = 2 \times 3 = 6$$

- B** 30 and 20

$$30 = 5 \times 2 \times 3$$

$$20 = 2 \times 2 \times 5$$

$$\text{GCF} = 2 \times 5 = 10$$

- C** 30 and 18

$$30 = 5 \times 2 \times 3$$

$$18 = 2 \times 3 \times 3$$

$$\text{GCF} = 2 \times 3 = 6$$

- D** 6 and 22

$$6 = 2 \times 3$$

$$22 = 2 \times 11$$

$$\text{GCF} = 2$$

- E** 42 and 58

$$42 = 7 \times 2 \times 3$$

$$58 = 2 \times 29$$

$$\text{GCF} = 2$$

- F** 63 and 36

$$63 = 3 \times 3 \times 7$$

$$36 = 2 \times 2 \times 3 \times 3$$

$$\text{GCF} = 3 \times 3 = 9$$

### Your Work

Find the GCF of 48, 50, and 75.  $48 = 2 \times 2 \times 2 \times 3 \times 2 \times 1$

$$50 = 5 \times 5 \times 2 \times 1$$

$$75 = 3 \times 5 \times 5 \times 1$$

$$\text{GCF} = 1$$



## — | (1-6) The Least Common Multiple (LCM)

The least common multiple (LCM) of a group of numbers is the lowest number that can be divided by each number in the group without leaving a remainder.

To find the LCM of the numbers 3, 6, and 8, list the multiples of each number and select the smallest number common to each.

**Step 1** Multiples of 3 are 3, 6, 9, 12, 15, 18, 21, **24**, 27, 30, etc.

**Step 2** Multiples of 6 are 6, 12, 18, **24**, 30, 36, etc.

**Step 3** Multiples of 8 are 8, 16, **24**, 32, etc.

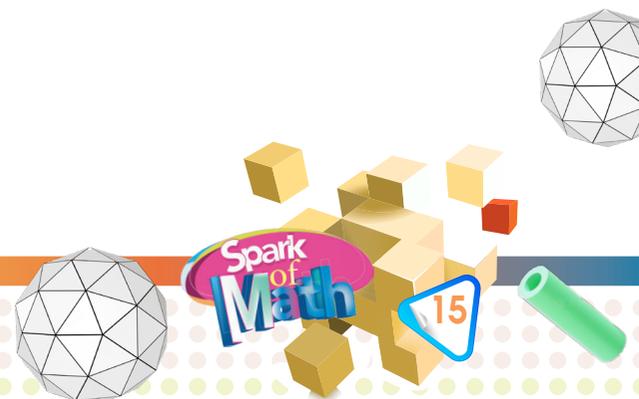
The LCM of the numbers 3, 6 and 8 is 24.

### 1. Find the LCM of 9 and 15.

Multiples of 9 are **9**, **18**, **27**, **36**, **45**, **54**, etc.

Multiples of 15 are **15**, **30**, **45**, **60**, **75**, **90**, etc.

The LCM is 45 .



## 2. Find the LCM of 10, 20, and 15.

Multiples of 10 are 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, etc.

Multiples of 20 are 20, 40, 60, 80, 100, 120, etc.

Multiples of 50 are 50, 100, 150, 200, 250, 300, etc.

The LCM is 100 .

### Your Work

#### 1. Find the LCM of 48 and 12.

Multiples of 12 are 12, 24, 36, 48, 60, etc.

Multiples of 48 are 48, 96, 144, etc.

The LCM is 48.

#### 2. Write down two numbers with a least common multiple of 30.

Students' own answers e.g, 10 and 15

#### 3. Discuss your answers.

Multiples of 10 are 10, 20, 30, 40, etc.

Multiples of 15 are 15, 30, 45, 60, etc.

The LCM is 30.



## (1-7) Problem Solving

1. A hall has two towers with rotating floodlights. Floodlight A takes 30 seconds to complete a revolution, while Floodlight B takes 25 seconds. At a certain moment, both floodlights shine at point C. When will both floodlights shine again simultaneously on point C?



Multiples of 30 are 30, 60, 90, 120, 150, etc.

Multiples of 25 are 25, 50, 75, 100, 125, 150, 175, etc.

The LCM is 150.

2. Sara has 3 types of ribbon of lengths: 84 cm, 140 cm, and 308 cm. She wishes to cut the ribbons into pieces of the same length for making butterfly knots.



- A If there is no unused ribbon, what is the longest length she can cut for each piece?

$$84 = 2 \times 2 \times 3 \times 7$$

$$140 = 2 \times 5 \times 2 \times 7$$

$$308 = 2 \times 2 \times 7 \times 11$$

$$\text{GCF} = 2 \times 2 \times 7 = 28$$

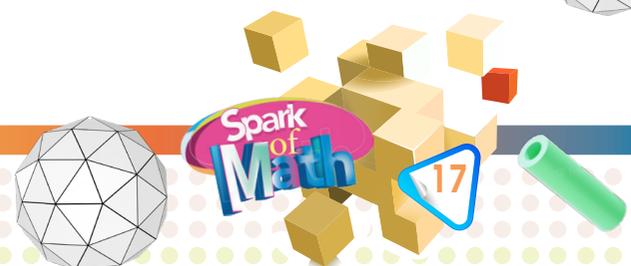
- B How many equal-length butterfly knot pieces will she get?

$$84 \div 28 = 3$$

$$140 \div 28 = 5$$

$$208 \div 28 = 11$$

$$3 + 5 + 11 = 19$$



## Show Your Turn

### 1. Circle the correct answers.

#### A A factor is:

- a. A number that you can multiply to get a product.
  - b. The result of multiplying a number by a whole number.
  - c. The greatest factor that two or more numbers have in common.
- 

#### B A multiple is:

- a. A number that you can multiply to get a product.
  - b. The result of multiplying a number by a whole number.
  - c. The greatest factor that two or more numbers have in common.
- 

#### C The factors of 12 are:

- a. 1, 2, 3, 6, and 12
- b. 1, 2, 3, 5, and 12
- c. 1, 2, 3, 4, 6, and 12

#### D The GCF of 27 and 30 is:

- a. 1
- b. 3
- c. 9

#### E The GCF of 12 and 18 is:

- a. 3
- b. 6
- c. 9

#### F The LCM of 3 and 5 is:

- a. 45
- b. 30
- c. 15



## 2. Find the answers, then check.

**A**

$$\begin{array}{r}
 \phantom{0}3\phantom{0}1 \\
 \phantom{0}1 \\
 \phantom{0}5\phantom{0}2 \\
 863 \\
 \times 538 \\
 \hline
 6,904 \\
 25,890 \\
 + 431,500 \\
 \hline
 464,294
 \end{array}$$

$$464,294 \div 538 = 863$$

**B**

$$\begin{array}{r}
 \phantom{0}1 \\
 \phantom{0}2\phantom{0}4 \\
 \phantom{0}2\phantom{0}4 \\
 6480 \\
 \times 256 \\
 \hline
 38,880 \\
 324,000 \\
 + 1,296,000 \\
 \hline
 1,658,880
 \end{array}$$

$$1,658,880 \div 6480 = 256$$

**C**

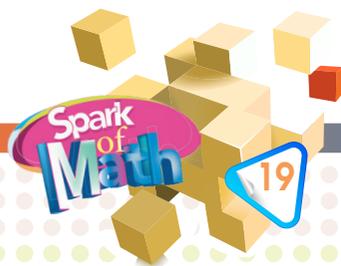
$$\begin{array}{r}
 \phantom{0}x\phantom{0}95 \\
 81 \overline{) 7695} \\
 \underline{- 729} \\
 405 \\
 \underline{- 405} \\
 0
 \end{array}$$

$$95 \times 81 + 0 = 7695$$

**D**

$$\begin{array}{r}
 \phantom{0}x\phantom{0}1308 \\
 47 \overline{) 61506} \\
 \underline{- 47} \\
 145 \\
 \underline{- 141} \\
 40 \\
 \underline{- 0} \\
 406 \\
 \underline{- 376} \\
 30
 \end{array}$$

$$1308 \times 47 + 30 = 61506$$





## Unit 2

# Fractions and Decimals

## Vocabulary



- fraction
- numerator
- denominator
- mixed number
- improper fraction
- simplest form
- reciprocal
- decimal
- convert

## Objectives



**Upon completion of this unit, you will be able to:**

- Multiply fractions and mixed numbers.
- Divide fractions and mixed numbers.
- Find the reciprocal of a fraction.
- Convert fractions and mixed numbers to decimals, and vice versa.
- Add and subtract decimals.
- Solve problems about fractions and decimals.

## (2-1) Multiplying Fractions

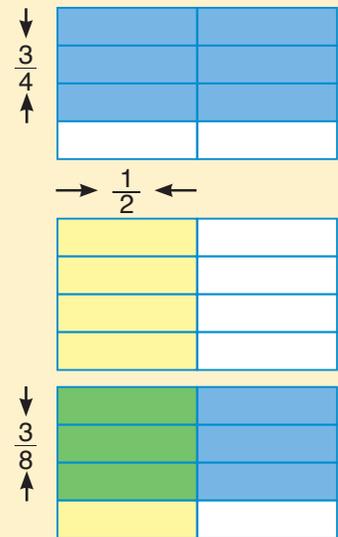
A fraction is a part of a whole that is divided into equal parts. A fraction can also be a part of a set.

To multiply fractions, multiply the numerators, followed by the denominators. Notice that the denominators do not have to be the same.

$$\frac{3}{4} \times \frac{1}{2} = \frac{3 \times 1}{4 \times 2} = \frac{3}{8}$$

Note that there are 3 parts of 4 colored blue and 1 part of 2 colored yellow.

Out of eight, there are three parts, each colored twice green.



To multiply mixed numbers, convert them to improper fractions, multiply the fractions, and simplify the result.

$$1\frac{3}{4} \times 2\frac{1}{2} = ?$$

$4 \times 1 + 3 = 7$   
 $2 \times 2 + 1 = 5$

$$\frac{7}{4} \times \frac{5}{2} = \frac{35}{8} = 4\frac{3}{8}$$

### 1. Multiply, and then write down in the simplest form.

$$\frac{1}{6} \times \frac{3}{4} = \frac{1 \times 3}{6 \times 4} = \frac{3}{24} = \frac{1}{8}$$

$$\frac{8}{10} \times \frac{7}{12} = \frac{8 \times 7}{10 \times 12} = \frac{56}{120} = \frac{7}{15}$$

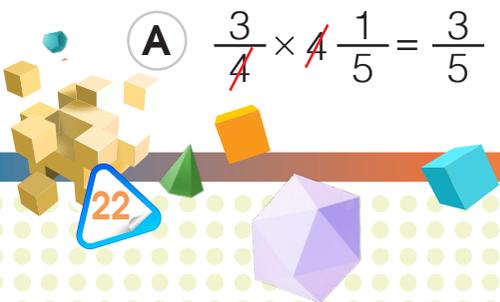
$$\frac{1}{5} \times 2\frac{7}{9} = \frac{1}{5} \times \frac{25}{9} = \frac{1 \times 25}{5 \times 9} = \frac{25}{45} = \frac{5}{9}$$

$$\frac{5}{4} \times 4\frac{1}{3} = \frac{5}{4} \times \frac{13}{3} = \frac{5 \times 13}{4 \times 3} = \frac{65}{12} = 5\frac{5}{12}$$

### 2. Write down "Yes" or "No" to indicate if the answers are correct, then discuss them with your classmates.

**A**  $\frac{3}{\cancel{4}} \times \cancel{4} \frac{1}{5} = \frac{3}{5}$  No

**B**  $\frac{\cancel{2}^1}{5} \times \frac{3}{\cancel{8}_4} = \frac{1 \times 3}{5 \times 4} = \frac{3}{20}$  Yes



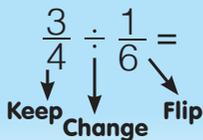
## (2-2) Dividing Fractions

Division is the inverse operation of multiplication. Therefore, dividing by a fraction can be done by multiplying it by its reciprocal.

The reciprocal of a fraction is the fraction turned upside down, where the numerator and denominator switch places.

$$\frac{3}{4} \div \frac{1}{6} =$$

- A** Keep the first fraction unchanged.
- B** Change the division sign to multiplication.
- C** Flip the second fraction (find the reciprocal).
- D** Multiply the numerators together.
- E** Multiply the denominators together.
- F** Simplify the resulting fraction.



$\frac{3}{4} \div \frac{1}{6} =$   
Keep   Change   Flip

$$\frac{3}{4} \times \frac{6}{1} = \frac{3 \times 6}{4 \times 1} = \frac{18}{4} = 4 \frac{2}{4}$$

**Here are a few examples of reciprocals:**

- The reciprocal of  $\frac{7}{5}$  is  $\longrightarrow \frac{5}{7}$ .
- The reciprocal of  $\frac{1}{2}$  is  $\longrightarrow \frac{2}{1}$ , or 2.
- To find the reciprocal of a mixed number, convert it to an improper fraction.

The reciprocal of  $3\frac{1}{5}$  is  $\longrightarrow \left(\frac{16}{5} = \frac{5}{16}\right)$ .

**1. Find the reciprocal of each fraction.**

**A**  $\frac{3}{8} \longrightarrow \frac{8}{3}$

**B**  $\frac{7}{4} \longrightarrow \frac{4}{7}$

**C**  $9 \longrightarrow \frac{1}{9}$



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## 2. Divide, and then write down in the simplest form.

**A**  $\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$

**B**  $\frac{4}{9} \div \frac{1}{2} = \frac{4}{9} \times \frac{2}{1} = \frac{8}{9}$

**C**  $\frac{5}{8} \div 1\frac{1}{4} = \frac{5}{8} \div \frac{5}{4} = \frac{5}{8} \times \frac{4}{5} = \frac{20}{40} = \frac{1}{2}$

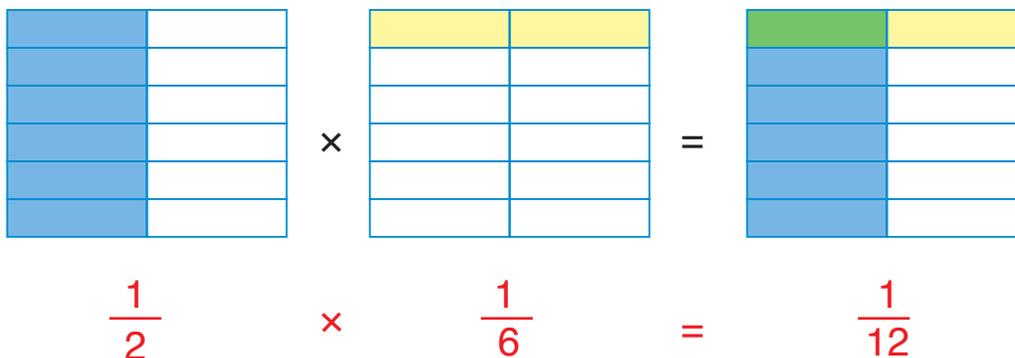
**D**  $\frac{6}{8} \div 2 = \frac{6}{8} \times \frac{1}{2} = \frac{6}{16} = \frac{3}{8}$

### Your Work

Use models to show that  $\frac{1}{2} \div 6 = \frac{1}{12}$ .

$$\frac{1}{2} \div 6 =$$

$$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$



## — (2-3) Fractions and Decimals

### Converting Fractions or Mixed Numbers to Decimals

To convert a fraction to a decimal, divide the numerator by the denominator until you get a decimal that terminates or repeats.

If it repeats, place a bar over the number that repeats.

#### Example 1:

$$\frac{7}{20} = 0.35$$
$$\begin{array}{r} \text{x0.35} \\ 20 \overline{) 7.00} \\ \underline{- 60} \phantom{0} \\ 100 \\ \underline{- 100} \\ 0 \end{array}$$

Note:  $7 = 7.0 = 7.00$

#### Example 2:

$$2\frac{7}{8} = 2.875$$
$$\begin{array}{r} \text{x0.875} \\ 8 \overline{) 7.000} \\ \underline{- 64} \phantom{00} \\ 60 \phantom{0} \\ \underline{- 56} \phantom{0} \\ 40 \\ \underline{- 40} \\ 0 \end{array}$$

Note: To convert a mixed number to a decimal, divide the fractional part to get a decimal, and then add the whole number.

#### Example 3:

(Repeating Decimal)

$$\frac{2}{3} = 0.\overline{6}$$
$$\begin{array}{r} \text{x0.66} \\ 3 \overline{) 2.00} \\ \underline{- 0} \phantom{00} \\ 20 \phantom{0} \\ \underline{- 18} \phantom{0} \\ 20 \\ \underline{- 18} \\ 2 \end{array}$$

Note: When the decimal result repeats, place a bar over the repeating digit(s).

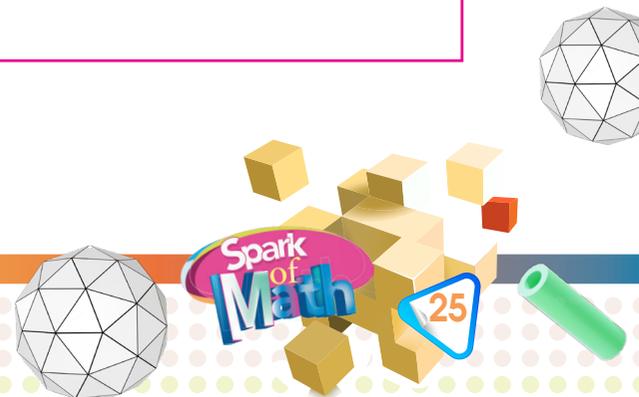
### 1. Convert these fractions to decimals.

**A**  $\frac{5}{8} = 0.625$

**B**  $6\frac{15}{1000} = 0.015$

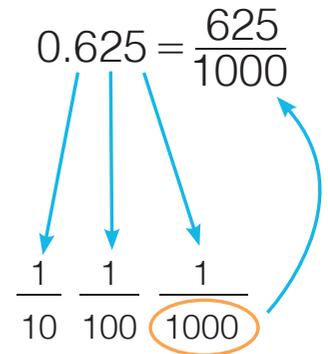
**C**  $\frac{9}{4} = 2.25$

**D**  $2\frac{3}{8} = 2.375$



## Converting Decimals to Fractions or Mixed Numbers

- A** Write down the decimal part as the numerator of the fraction without the decimal point.
- B** Identify the decimal place value of the last digit. This becomes the denominator of the fraction.
- C** Simplify the fraction, if possible.



$$\frac{625 \div 5}{1000 \div 5} = \frac{125 \div 5}{200 \div 5} = \frac{25 \div 5}{40 \div 5} = \frac{5}{8}$$

If the decimal contains a whole number, retain the whole part, convert the decimal part to a fraction, and then combine both to obtain the mixed fraction.

$$2.36 = 2 \frac{36 \div 4}{100 \div 4} = 2 \frac{9}{25}$$

### 1. Convert these decimals to fractions or mixed numbers.

$$0.1 = \frac{1}{10}$$

$$0.24 = \frac{24}{100} = \frac{6}{25}$$

$$0.71 = \frac{71}{100}$$

$$2.705 = 2 \frac{705}{1000} = 2 \frac{141}{200}$$

### Your Work

- Show that  $\frac{3}{4} = 0.75$  using models. *Students' own answers*
- Explain why  $(0.2 = 0.20 = 0.200)$ . *Zeros to the right of the last non-zero digit do not change the actual value of the number.*
- Convert  $\frac{3}{8}$  to a decimal number.  $\frac{3}{8} = 0.375$



## (2-4) Adding and Subtracting Decimals

**To add or subtract decimal numbers:**

- Align numbers vertically by place value, carefully lining up the decimal points.
- Add zeros to the empty digits.
- Add or subtract the numbers, starting from the right.
- Drag the decimal point straight down.

### Examples:

●  $14 + 3.87 + 4.762 = \rightarrow 14.000$  ( $14 = 14.0 = 14.000$ )

$$\begin{array}{r} 14.000 \\ + 3.870 \\ + 4.762 \\ \hline 22.632 \end{array}$$

●  $19.55 - 7.4 = \rightarrow 19.55$  ( $7.4 = 7.40$ )

$$\begin{array}{r} 19.55 \\ - 7.40 \\ \hline 12.15 \end{array}$$

**1. Add or subtract.**

A  $2.84 + 17.5 =$

$$\begin{array}{r} 1 \\ 2.84 \\ + 17.5 \\ \hline 20.34 \end{array}$$

B  $29.3 - 4.781 =$

$$\begin{array}{r} 129 \\ 29.300 \\ - 4.781 \\ \hline 24.519 \end{array}$$



## 2. Add or subtract.

$$\begin{array}{r} 1 \\ 5.6 \\ + 5.4 \\ \hline 11.0 \end{array}$$

$$\begin{array}{r} 1 \quad 1 \\ 79.195 \\ + 6.721 \\ \hline 85.916 \end{array}$$

$$\begin{array}{r} 1 \quad 1 \\ 4.770 \\ - 0.291 \\ \hline 5.061 \end{array}$$

$$\begin{array}{r} 1 \quad 1 \\ 2040.300 \\ - 1572.914 \\ \hline 3613.214 \end{array}$$

## 3. Circle "Yes" or "No" to indicate if the answers are correct, then discuss them with your classmates.

**A**  $45.63 + 78.2 =$

$$\begin{array}{r} 4563 \\ + 782 \\ \hline 5345 \end{array} \quad (\text{Yes} / \text{No})$$

Adding numbers without considering the decimal points will give wrong results.

**B**  $37.41 + 5.9 =$

$$\begin{array}{r} 37.41 \\ + 5.09 \\ \hline 42.50 \end{array} \quad (\text{Yes} / \text{No})$$

Adding a zero between the decimal point and the digits shifts the place value of the numbers.

### Your Work

Students' own answers

1. Use models to find the answer to  $(3.5 + 1.16)$ .

2. Find  $(0.75 - 0.04)$  using fractions, and write down the outcome in the simplest form.

$$\frac{75}{100} - \frac{4}{100} = \frac{71}{100} = 0.71$$



## — (2-5) Problem Solving

1. Ismail has 8 bags of sweets. His younger brother ate 1.5 bags, and he ate 2 bags. How many bags of sweets are left?

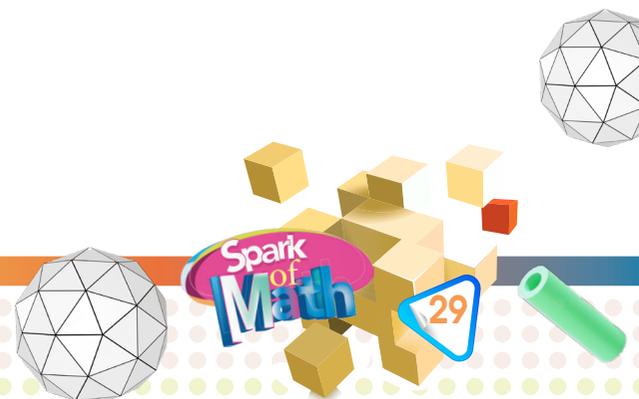
$$8 - (1.5 + 2) =$$

$$8 - 3.5 =$$

$$4.5$$

2. At a Jordanian school,  $\frac{4}{5}$  of the students take foreign languages. Of those students,  $\frac{3}{4}$  take Spanish. What fraction represents the students who take Spanish out of all the students taking foreign languages?

$$\frac{4}{5} \times \frac{3}{4} = \frac{4 \times 3}{5 \times 4} = \frac{12}{20} = \frac{3}{5}$$





# Show Your Turn

1. Find the reciprocal of each fraction.

A  $\frac{2}{7} \rightarrow \frac{7}{2}$

B  $3\frac{1}{4} \rightarrow \frac{13}{4} \rightarrow \frac{4}{13}$

C  $11 \rightarrow \frac{1}{11}$

2. Find.

A  $3\frac{8}{10} \times 2\frac{1}{2} =$

$$\frac{38}{10} \times \frac{5}{2} = \frac{38 \times 5}{10 \times 2} = \frac{190}{20} = 9\frac{1}{2}$$

B  $3\frac{1}{5} \div \frac{4}{5} =$

$$\frac{16}{5} \div \frac{4}{5} = \frac{16}{5} \times \frac{5}{4} = \frac{16}{4} = 4$$

C  $32.6 + 4.17 =$

$$\begin{array}{r} 32.60 \\ + 4.17 \\ \hline 36.77 \end{array}$$

D  $29 - 5.42 =$

$$\begin{array}{r} 29.00 \\ - 5.42 \\ \hline 23.58 \end{array}$$

3. Convert these decimals to fractions or mixed numbers, then simplify.

A  $3.6 =$

$$3\frac{6}{10}$$

$$3\frac{3}{5}$$

B  $4.17 =$

$$4\frac{17}{100}$$

C  $11.125 =$

$$11\frac{125}{1000}$$

$$11\frac{1}{8}$$





## Vocabulary



- metric units
- rectangular prism
- volume
- circle
- center
- radius
- diameter
- acute angle
- obtuse angle
- right angle
- straight angle
- protractor
- complementary angles
- supplementary angles
- vertical angles

## Objectives



**Upon completion of this unit, you will be able to:**

- Add and subtract different metric units of length.
- Measure the volume of a rectangular prism.
- Identify the different parts of a circle, including the center, radius, and diameter.
- Find the diameter and radius of a circle.
- Describe and identify the relationship between angle pairs.
- Find missing angle measurements in pairs.

## (3-1) Adding and Subtracting Metric Units

Different metric units measure various lengths and distances accurately. Millimeters, centimeters, and decimeters measure shorter lengths, while meters and kilometers measure longer distances. When adding or subtracting metric units, ensure that the units are the same.

To add  $1900 \text{ cm} + 23 \text{ m} + 25.9 \text{ m}$ :

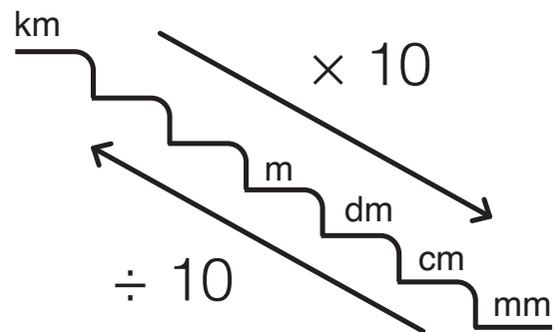
### Step 1

Convert all units to the same unit.

To convert  $1900 \text{ cm}$  to meters, divide  $1900 \text{ cm}$  by  $100$ , which equals  $19 \text{ m}$ .

$23$  is already in meters.

$25.9$  is already in meters.



### Step 2

Add the values in meters.

$$19 \text{ m} + 23 \text{ m} + 25.9 \text{ m} = 67.9 \text{ m}$$

$$\begin{array}{r} 19.0 \\ 23.0 \\ +25.9 \\ \hline 67.9 \end{array}$$



## 1. Add or subtract.

**A**  $34 \text{ m} + 8269 \text{ mm} = 34 \text{ m} \times 1000 = 34000 \text{ mm}$   
 $34000 \text{ mm} + 8269 \text{ mm} = 42269 \text{ mm}$

42269

mm

**B**  $42 \text{ km} - 314 \text{ dm} = 42 \text{ km} \times 1000 = 42000 \text{ m}$   
 $314 \text{ dm} \div 10 = 31.4 \text{ m}$   
 $42000 \text{ m} - 31.4 \text{ m} = 41968.6 \text{ m}$

41968.6

m

**C**  $3 \text{ km} + 15 \text{ m} - 2 \text{ dm} = 3 \text{ km} \times 1000 = 3000 \text{ m}$   
 $2 \text{ dm} \div 10 = 0.2 \text{ m}$   
 $3000 \text{ m} + 15 \text{ m} - 0.2 \text{ m} = 3014.8 \text{ m}$

3014.8

m

**D**  $6500 \text{ mm} + 17 \text{ cm} = 6500 \text{ mm} \div 100 = 65 \text{ dm}$   
 $17 \text{ cm} \div 10 = 1.7 \text{ dm}$   
 $65 \text{ dm} + 1.7 \text{ dm} = 66.7 \text{ dm}$

66.7

dm

### Your Work

What is the sum of  $3 \text{ dm}^2$  and  $15 \text{ cm}^2$ ?  $3 \text{ dm}^2 \times 100 = 300 \text{ cm}^2$

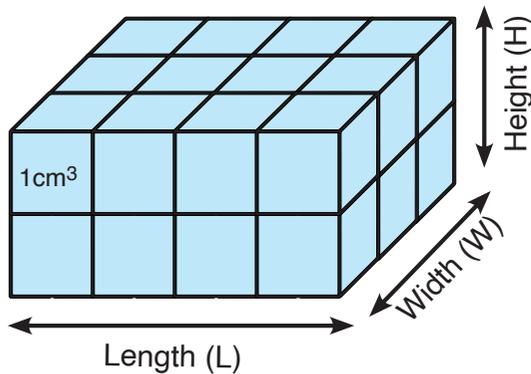
Discuss your answer.

$$300 \text{ cm}^2 + 15 \text{ cm}^2 = 315 \text{ cm}^2$$



## (3-2) Rectangular Prism Volume

A rectangular prism has three dimensions: length, width, and height. It also has six faces. The prism's height reveals the number of cube layers stacked within it.



$$\begin{aligned} \text{Length (L)} &= 4 \text{ cm} \\ \text{Width (W)} &= 3 \text{ cm} \\ \text{Height (H)} &= 2 \text{ cm} \\ \mathbf{V} &= \mathbf{24 \text{ cm}^3} \end{aligned}$$

Each cube in the prism has a volume of  $1 \text{ cm}^3$ . Given that the height is 2 cm, the length is 4 cm, and the width is 3 cm, the total volume is calculated by multiplying these dimensions:  $2 \times 3 \times 4 = 24 \text{ cm}^3$ .

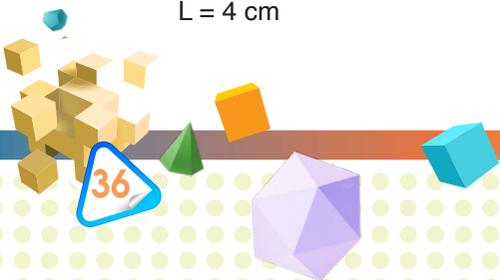
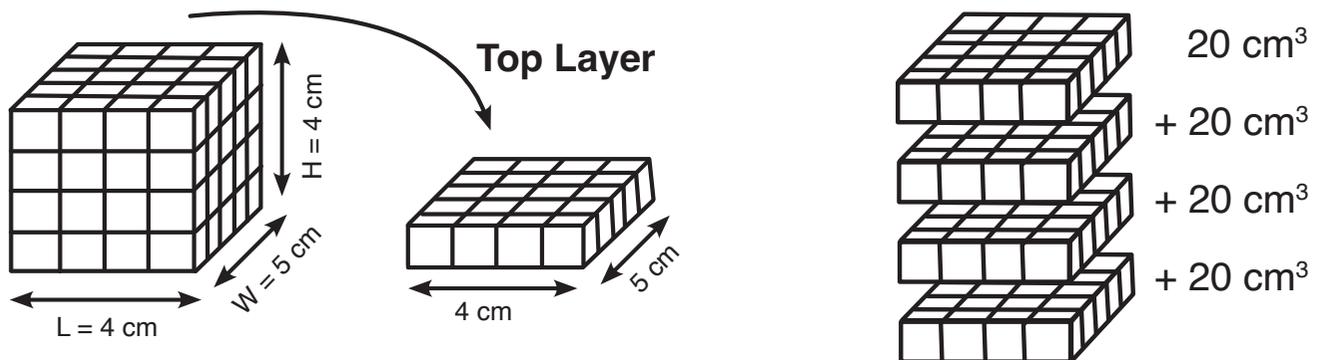
Therefore, the volume of any rectangular prism can be found using the formula:

$$\mathbf{\text{Volume(V)} = \text{Length} \times \text{Width} \times \text{Height}}$$

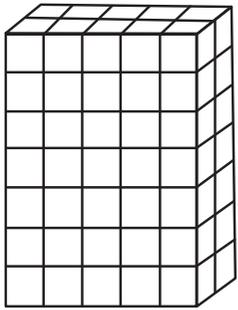
You can calculate the volume of a rectangular prism by counting the cubes in one layer and then multiplying that number by the number of layers.

For example, if the top layer of the prism is composed of four rows of five  $1 \text{ cm}^3$  blocks, then the volume of that top layer is  $4 \times 5 = 20 \text{ cm}^3$ .

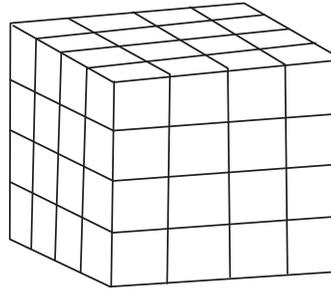
If there are 4 layers, each with a volume of  $20 \text{ cm}^3$ , then the total volume of the prism is  $4 \times 20 = 80 \text{ cm}^3$ .



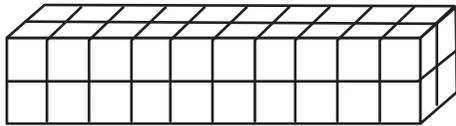
**1. Find the dimensions and volume of each rectangular prism.**



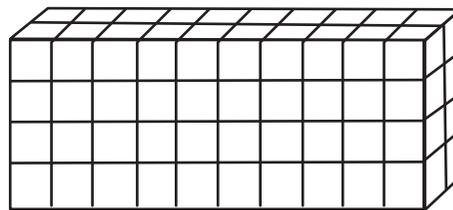
L = 5 cm  
 W = 2 cm  
 H = 7 cm  
 V = 70 cm<sup>3</sup>



L = 4 cm  
 W = 4 cm  
 H = 4 cm  
 V = 64 cm<sup>3</sup>

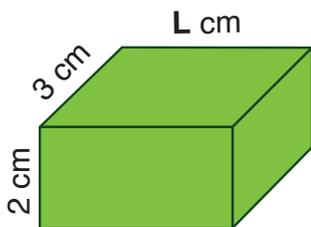


L = 10 cm  
 W = 2 cm  
 H = 2 cm  
 V = 40 cm<sup>3</sup>

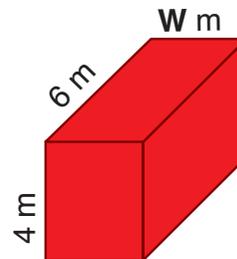


L = 10 cm  
 W = 2 cm  
 H = 4 cm  
 V = 80 cm<sup>3</sup>

**2. Find the values of the missing dimensions.**



If the volume of the prism is 30 cm<sup>3</sup>, L = 5 cm.



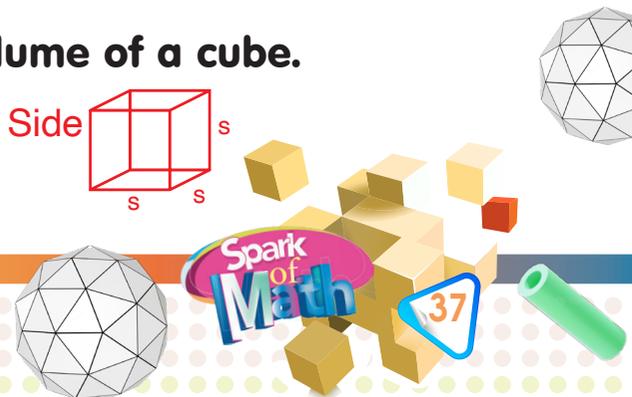
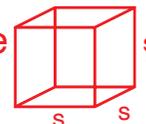
If the volume of the prism is 72 m<sup>3</sup>, W = 3 m.

**Your Work**

**Write down the formula to calculate the volume of a cube. Then, show your work.**

Volume = Side x Side x Side

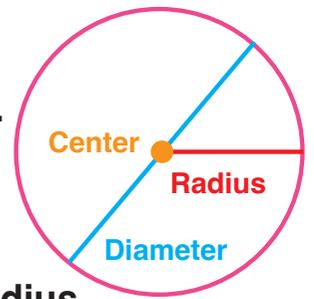
$V = S \times S \times S$



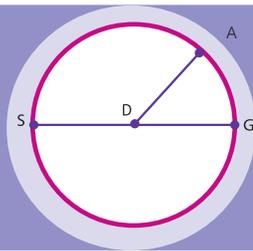
## (3-3) Circles

Circles are two-dimensional shapes with several key parts: the center, radius, and diameter.

- **Center:** It is the middle point at which a circle gets its name.
- **Radius:** It is a straight line from the center to any point on the circle's edge.
- **Diameter:** It is a straight line passing through the center from one side of the circle to the other, equal to twice the radius.

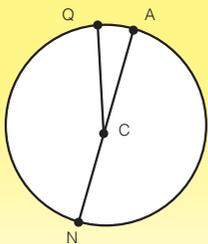


**Example:**

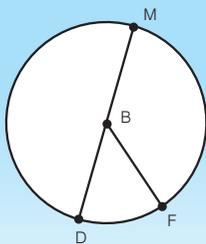


Name of Circle: D  
 Radius of the Circle: DA  
 Diameter of the Circle: SG

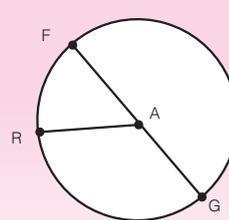
**1. Write down the name of each circle, radius, and diameter.**



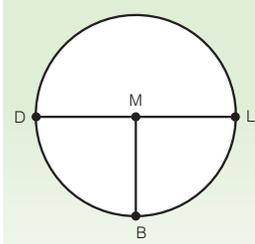
Circle: **C**  
 Radius: **CQ**  
 Diameter: **AN**



Circle: **B**  
 Radius: **BF**  
 Diameter: **MD**



Circle: **A**  
 Radius: **AR**  
 Diameter: **FG**



Circle: **M**  
 Radius: **MB**  
 Diameter: **DL**

**To find the diameter, use this formula:**

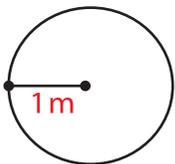
$$\text{Diameter} = 2 \times \text{Radius}$$

$$D = 2 \times R$$

**Example:** A bicycle wheel has a radius of 311 millimeters. What is the diameter of the wheel?

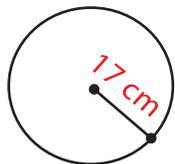
$$D = 2 \times R \longrightarrow D = 2 \times 311 \text{ mm} \longrightarrow D = 622 \text{ mm}$$

**2. Find the radius and diameter values of each circle.**



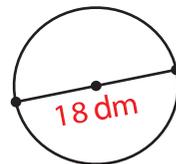
Radius = 1 m

Diameter =  $\frac{2 \times 1}{2} =$  2 m



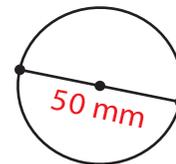
Radius = 17 cm

Diameter =  $\frac{17 \times 2}{2} =$  34 cm



Radius = 9 dm

Diameter =  $\frac{2 \times 9}{2} =$  18 dm



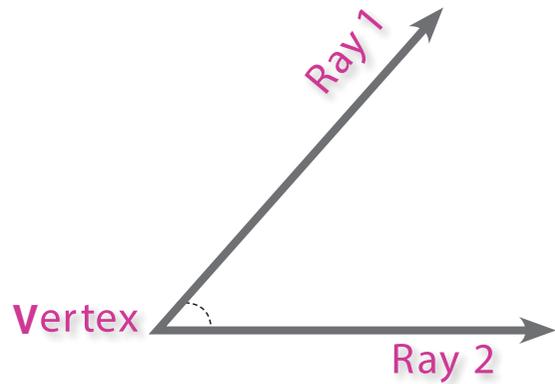
Radius = 25 mm

Diameter =  $\frac{25 \times 2}{2} =$  50 mm



## (3-4) Measuring Angles

An angle is formed by two rays that share a common endpoint, known as the vertex. Angles are measured in units called degrees. There are four types of angles: acute, right, obtuse, and straight.



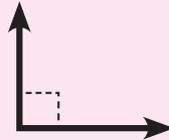
● **Acute Angle:** An angle that is less than  $90^\circ$ .



● **Obtuse Angle:** An angle that is between  $90^\circ$  and  $180^\circ$ .



● **Right Angle:** An angle that is exactly  $90^\circ$ .



● **Straight Angle:** An angle that is exactly  $180^\circ$ .



## The Protractor

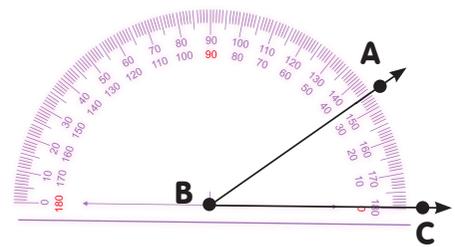
The protractor serves as a tool for measuring angles. To do so, follow these steps:

**Step 1** Place the center point of the protractor on the vertex of the angle.

**Step 2** Align the 0 mark of the protractor with one side of the angle.

**Step 3** Read the angle measurement where the other side intersects the protractor.

The measure of the adjacent angle is  $35^\circ$ .



### 1. Use a protractor to measure the given angles.

$$\angle AOC = \underline{\quad 60^\circ \quad}$$

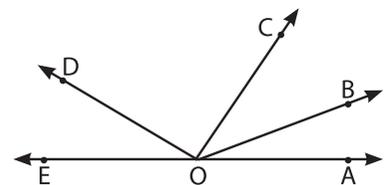
$$\angle COE = \underline{\quad 125^\circ \quad}$$

$$\angle BOD = \underline{\quad 130^\circ \quad}$$

$$\angle DOE = \underline{\quad 30^\circ \quad}$$

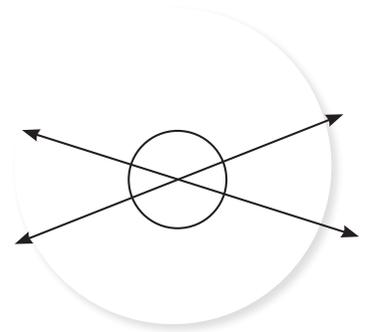
$$\angle AOB = \underline{\quad 20^\circ \quad}$$

$$\angle AOD = \underline{\quad 150^\circ \quad}$$



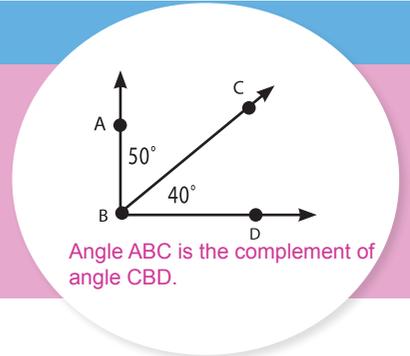
## ● Pairs of Angles

In geometry, pairs of angles can relate to each other in several ways. When two or more lines or rays intersect, they form pairs of angles. These pairs can be categorized into three main types: complementary angles, supplementary angles, and vertical angles.



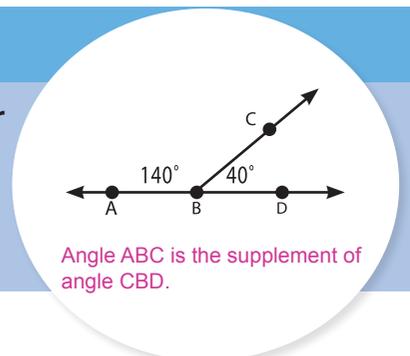
### ● Complementary Angles

Two angles are called complementary if the sum of their measurements equals  $90^\circ$  (a right angle). Each angle in the pair is referred to as the complement of the other.



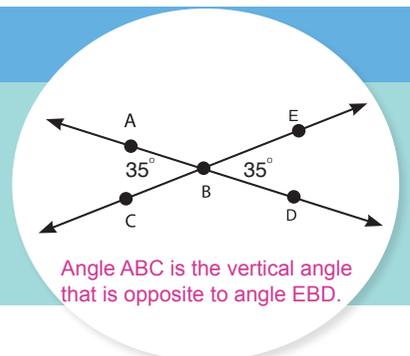
### ● Supplementary Angles

Two angles are called supplementary if the sum of their measurements equals  $180^\circ$  (a straight angle). Each angle in the pair is referred to as the supplement of the other.



### ● Vertical Angles

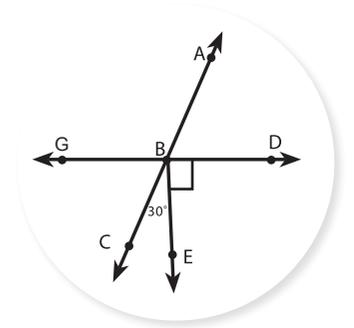
When two lines intersect, they form two pairs of angles. The angles that are opposite each other at the intersection are called vertical angles. Vertical angles are always equal to each other.



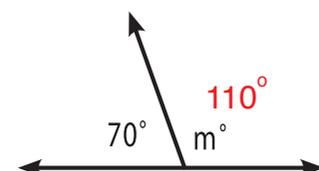
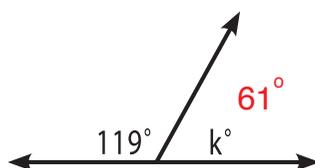
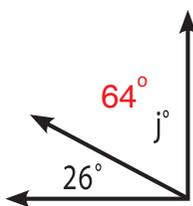
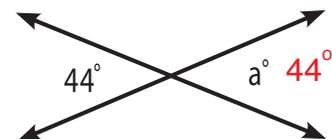
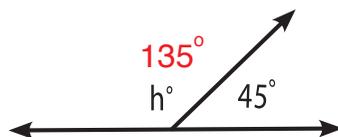
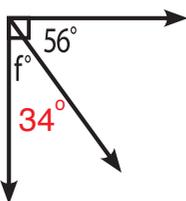
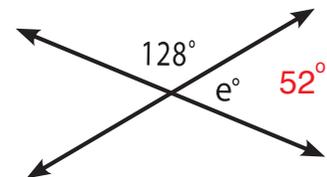
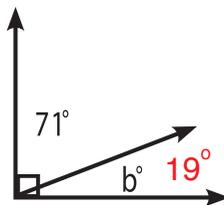
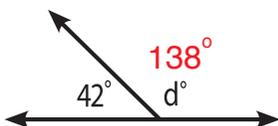
## Example:

Calculate the measurements of the angles based on the relationship between the angle pairs.

- $\angle DBE$  is a right angle because of the square corner symbol.
- Note that  $\angle GBC$  and  $\angle CBE$  are complementary angles.  
So,  $\angle GBC = \angle GBE - \angle CBE = 90^\circ - 30^\circ = 60^\circ$ .
- Note that  $\angle GBA$  and  $\angle GBC$  are supplementary angles.  
So,  $\angle GBA = 180^\circ - \angle GBC = 180^\circ - 60^\circ = 120^\circ$ .
- Note that  $\angle GBC$  and  $\angle ABD$  are vertical angles.  
So,  $\angle ABD = \angle GBC = 60^\circ$ .



1. Calculate each angle's measurement based on the relationship between the angle pairs.



## (3-5) Problem Solving

1. Bashar rode 2 kilometers on his bike, while his sister Lana rode 3,000 meters. Who rode farther, and by how much?



Lana  $3,000\text{m} = 3\text{ km}$   $3\text{ km} > 2\text{ km}$

2. Manal has a box with a length of 12 cm, a height of 8 cm, and a width of 6 cm. What is the volume of the box?



$12\text{ cm} \times 8\text{ cm} \times 6\text{ cm} = 576\text{ cm}^3$

3. Omar is 143 centimeters tall, and Laila is 1.35 meters tall. Who is taller?



Omar  $143\text{ cm} = 1.43\text{ m}$   $1.43\text{ m} > 1.35\text{ m}$

4. Maya bought a fish tank with the following measurements: length 50 cm, width 30 cm, and height 4 cm. What is the fish tank's volume?



$50\text{ cm} \times 30\text{ cm} \times 4\text{ cm} = 6,000\text{ cm}^3$



## Show Your Turn

### 1. Add or subtract.

$$5094 \text{ m} + 3 \text{ km} = \underline{8044 \text{ m}}$$

$$5094 \text{ m} + 3000 \text{ m} =$$

$$4.9 \text{ m}^2 + 5 \text{ dm}^2 = \underline{495 \text{ dm}^2}$$

$$490 \text{ dm}^2 + 5 \text{ dm}^2 =$$

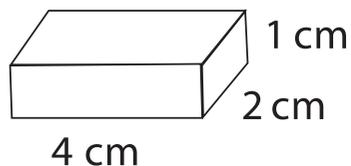
$$0.5 \text{ m} - 7 \text{ cm} = \underline{43 \text{ cm}}$$

$$50 \text{ cm} - 7 \text{ cm} =$$

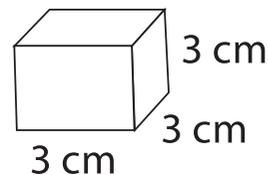
$$824 \text{ cm}^2 - 4000 \text{ mm}^2 = \underline{784 \text{ cm}^2}$$

$$824 \text{ cm}^2 - 40 \text{ cm}^2 =$$

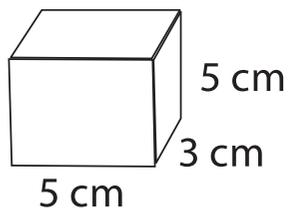
### 2. Find the volume of each rectangular prism.



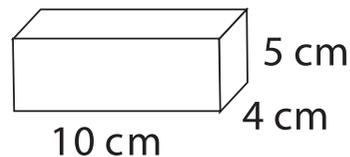
$$V = \underline{1 \times 2 \times 4 = 8 \text{ cm}^3}$$



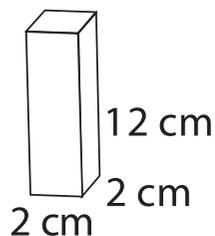
$$V = \underline{3 \times 3 \times 3 = 27 \text{ cm}^3}$$



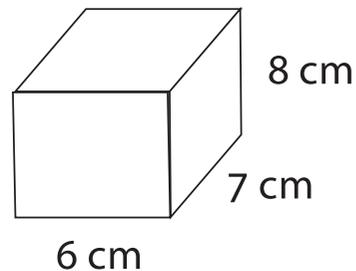
$$V = \underline{5 \times 5 \times 3 = 75 \text{ cm}^3}$$



$$V = \underline{5 \times 4 \times 10 = 200 \text{ cm}^3}$$



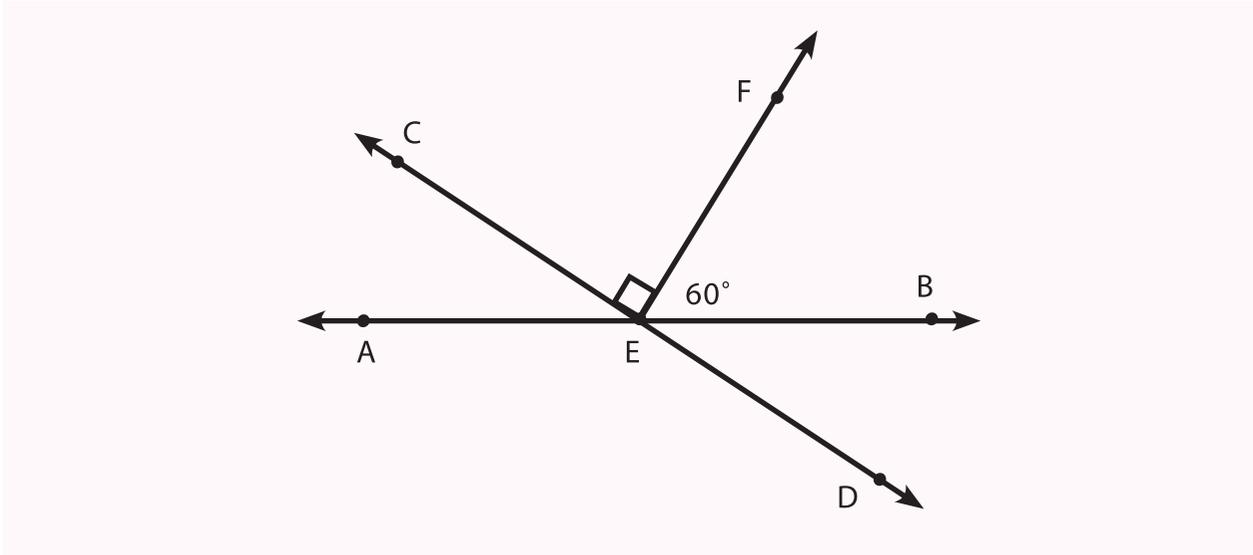
$$V = \underline{12 \times 2 \times 2 = 48 \text{ cm}^3}$$



$$V = \underline{6 \times 7 \times 8 = 336 \text{ cm}^3}$$



3. Look at the figure and answer the questions. The boxes below provide some of the answers.



straight

protractor

right

angle

degrees

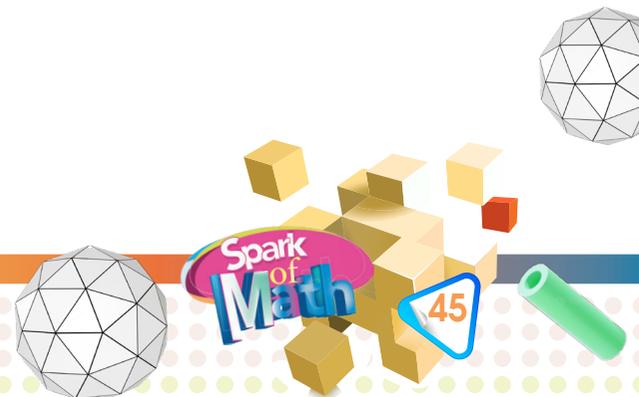
obtuse

acute

- (A) The figure formed by two rays that share a common endpoint is called an angle.
- (B) The basic unit for measuring angles is the degrees.
- (C) The instrument used to measure angles is a protractor.
- (D) The angle at vertex AEC is acute.
- (E) An angle whose measure is between  $0^\circ$  and  $90^\circ$  is an acute angle.
- (F) An angle whose measure is between  $90^\circ$  and  $180^\circ$  is an obtuse angle.



- G Angle CEF has a measure of  $90^\circ$  and is called a right angle.
- H Angle CED is called a straight angle.
- I Two acute angles in the figure are  $\angle$  DEB and  $\angle$  AEC.
- J Two complementary angles in the figure are BEF and BED.
- K Two supplementary angles in the figure are CEF and FED.
- L Two vertical angles in the figure are CEA and BED.



$$-4 = x - 2$$

$$2x + y = x + 2$$

$$2x = x + 2 - y$$

$$x = 2 - y$$



$$\sin(-a) = -\sin a$$

$$(12-a) * (4+b) = 20$$

$$12-a = \frac{20}{(4+b)}$$

$$12-a = \frac{5}{b}$$

$$12b - ab = 5$$

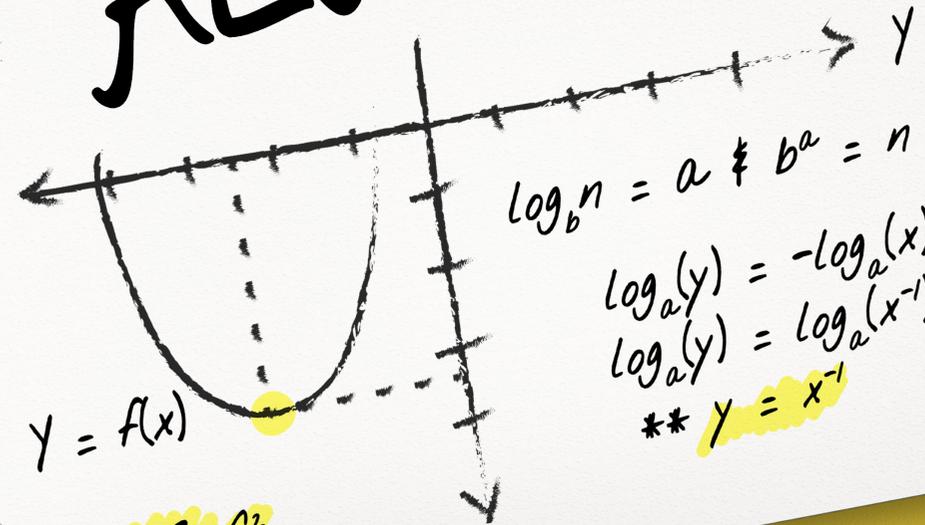
$$12b = 5 -$$

# ALGEBRA



$$y = b^x$$

$$x = \log_b y$$

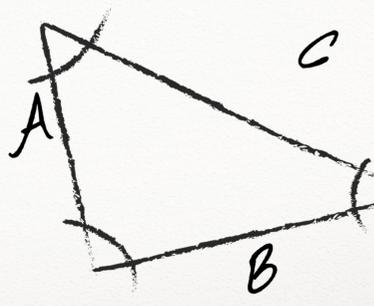
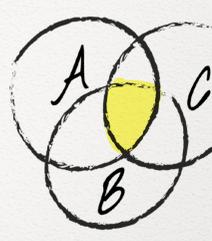


$$\log_b n = a \quad \& \quad b^a = n$$

$$\log_a(y) = -\log_a(x)$$

$$\log_a(y) = \log_a(x^{-1})$$

$$** y = x^{-1}$$



$$y = f(x)$$

$$A^2 + B^2 = C^2$$

## Unit 4

# Algebra

## Vocabulary



- algebra
- variable
- expression
- equation

## Objectives



**Upon completion of this unit, you will be able to:**

- Identify algebra.
- Differentiate between an expression and an equation.
- Evaluate algebraic expressions.
- Solve algebraic equations.

## — | (4-1) Introduction to Algebra

Algebra is a type of mathematics that uses variables to represent quantities. Variables are symbols, usually letters, that represent unknown or changeable values in mathematical expressions and equations. They allow us to generalize problems and solve for unknown quantities.

### Example:

Juan has  $X$  dollars. She wants to buy a CD that costs 11 dollars. To determine how much additional money she needs, we can use the following formula:

$$11 - X = Y$$

$Y$  is the amount of money Juan needs. If Juan possesses 4 dollars, we can replace  $X$  with 4.

$$11 - 4 = 7$$

So, Juan needs 7 more dollars.

$11 - X$  is an expression. An expression is a phrase that includes at least one variable.

$11 - X = Y$  is an equation. An equation is an algebraic or numerical sentence that shows two quantities are equal.

### 1. Write down an algebraic expression for each word phrase.

The Phrase	The Expression
Adding $Y$ to number 8	$Y + 8$
Subtracting 14 from $X$	$X - 14$
The product of 4 and $A$	$4A$
30 more than $S$	$S + 30$
Subtracting 5 from 4 times $C$	$4C - 5$
30 divided by $C$	$30 \div C$
8 times the product of 5 and $X$	$8(5X)$
15 less than $V$	$V - 15$



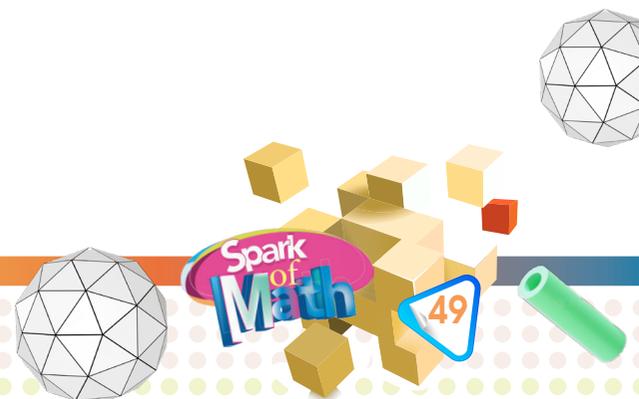
## 2. Circle the correct answers.

$13 = 5 + K$	(expression / <b>equation</b> )
$4S + 9$	( <b>expression</b> / equation)
$5A + 17 + 3B = 96$	(expression / <b>equation</b> )
$34Y - 6 = 44$	(expression / <b>equation</b> )
$7P$	( <b>expression</b> / equation)
$4A - 3D$	( <b>expression</b> / equation)
$31 = X - 18$	(expression / <b>equation</b> )

### Your Work

Write down a formula to find the perimeter ( $P$ ) of a square with side length ( $L$ ).

$$P = 4L$$



## (4-2) Evaluating Algebraic Expressions

Evaluating algebraic expressions entails substituting specific values for the variables in the expression, followed by the arithmetic operations.

To evaluate the expression  $12 + 3Y$  when  $Y = 2$ :

### Step 1

Substitute the value of  $Y$  into the expression.

$$12 + 3Y =$$

$$12 + 3 \times 2 =$$

### Step 2

Perform the equation according to the order of operations.

$$12 + 3 \times 2 =$$

$$12 + 6 = 18$$

**1. Evaluate each expression according to the given value of each variable.**

**(A)**  $X - 9$ , when  $X = 11$   $11 - 9 = 2$

**(B)**  $2W + 7$ , when  $W = 10$   $2 \times 10 + 7 = 27$

**(C)**  $P^2 + 5$ , when  $P = 6$   $6 \times 6 + 5 = 41$

**(D)**  $10 - 3F$ , when  $F = 3$   $10 - 3 \times 3 = 1$

**(E)**  $(15 - X) \times 2$ , when  $X = 5$   $(15 - 5) \times 2 = 20$

**(F)**  $6 + G$ , when  $G = 8$   $6 + 8 = 14$

### Your Work

**1. Find the value of  $K$  that makes the expression  $(2K)$  less than 7.**

**Discuss your answer.**

$$\frac{2}{2}K < \frac{7}{2} \quad K < 3.5$$

**2. How much greater is the perimeter of a square with a side length of 6 cm than a square with a side length of 5 cm?**

$$P_1 = 6 \times 4 = 24 \text{ cm}$$

$$P_2 = 5 \times 4 = 20 \text{ cm}$$

$$P_1 - P_2 = 24 - 20 = 4 \text{ cm}$$



## (4-3) Solving Algebraic Equations

Solving algebraic equations entails finding the value of the variable(s) that make the equation true.

To solve the equation  $Y - 3 = 5$ :

### Step 1

Isolate the variable  $Y$  by itself on one side of the equation.

For the equation  $Y - 3 = 5$ , isolate  $Y$ .

### Step 2

Perform inverse operations on both sides. To maintain equality, perform the same operation on both sides. Addition is the opposite of subtraction. Add 3 to both sides of the equation.

$$Y - \cancel{3} = 5$$

$+3 \quad +3$

### Step 3

Perform the equation according to the order of operations.

$$Y = 5 + 3$$

$$Y = 8$$

So, the solution to the equation  $Y - 3 = 5$  is  $Y = 8$ .

### 1. Solve the equations.

**A**

$$X + \cancel{7} = \cancel{18}$$

$\cancel{-7} \quad \cancel{-7}$

$$X = 18 - 7$$
$$X = 11$$

**B**

$$A - \cancel{15} = 22$$

$\cancel{+15} \quad \cancel{+15}$

$$A = 22 + 15$$
$$A = 37$$

**C**

$$\cancel{3}Y = \cancel{6}$$

$\cancel{\div 3} \quad \cancel{\div 3}$

$$Y = 6 \div 3$$
$$Y = 2$$

**D**

$$C \div \cancel{3} = 6$$

$\cancel{\times 3} \quad \cancel{\times 3}$

$$C = 6 \times 3$$
$$C = 18$$

### Your Work

Solve the equation  $2W + 1 = 9$ , and then check your answer.

$$\cancel{2} W = \frac{\cancel{8}}{\cancel{2}} \quad W = 8 \div 2 \quad W = 4$$

$\cancel{-1} \quad \cancel{-1}$



## (4-4) Problem Solving

Circle the equation that represents each situation, then find its solution.

1. There are 18 tennis balls in a bucket. There are  $X$  blue tennis balls and 6 yellow balls.

The correct equation that describes this situation is:

- A  $X - 6 = 18$        B  $X + 6 = 18$   
 C  $18 + X = 6$        D  $6X - 4 = 18$

The solution is:  $\frac{X + 6 = 18}{-6 \quad -6} \quad \frac{X = 18 - 6}{X = 12}$

2. Haitham has  $Y$  amount of dinars in his pocket. He spent 3 JD on a chocolate bar, leaving him with 8 JD.

The correct equation that describes this situation is:

- A  $3 - Y = 8$        B  $Y + 3 = 8$   
 C  $8 - 3 = Y$        D  $Y - 3 = 8$

The solution is:  $\frac{Y - 3 = 8}{+3 \quad +3} \quad \frac{Y = 8 + 3}{Y = 11}$

3. Ali has  $W$  golf tees in his bag. His sister has 4 times as many tees as Ali does and has 36 tees in total.

The correct equation that describes this situation is:

- A  $\frac{W}{4} = 36$        B  $\frac{4}{W} = 36$   
 C  $36W = 4$        D  $4W = 36$

The solution is:  $\frac{4}{4} W = \frac{36}{4}$

$$W = 36 \div 4$$

$$W = 9$$

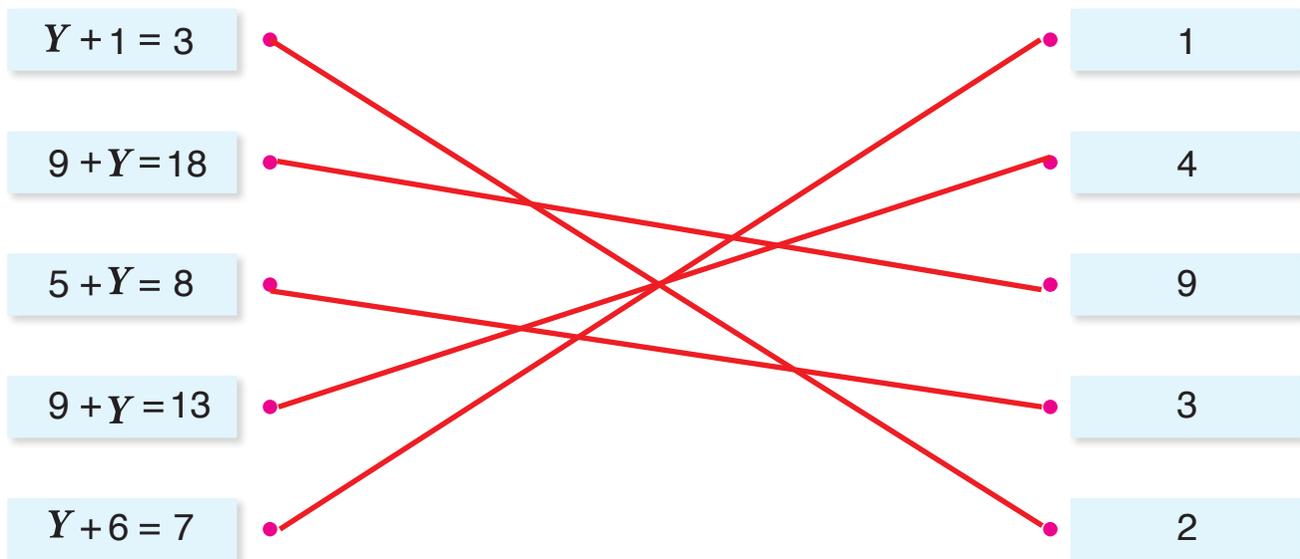


# Show Your Turn

1. Write down whether each item below is an equation or expression.

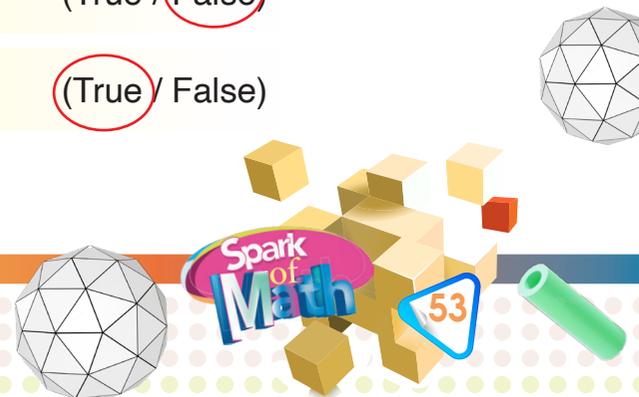
- (A)  $9 - N = 15$  equation
- (B)  $9B$  expression
- (C)  $2C = 48 + A$  equation

2. Match each equation to its corresponding  $Y$  value.



3. Circle the correct answers.

- (A)  $A - 6 = 4 \longrightarrow A = 10$  (True / False)
- (B)  $\frac{C}{12} = 12 \longrightarrow C = 6$  (True / False)
- (C)  $\frac{30}{5} = H \longrightarrow H = 6$  (True / False)
- (D)  $8Z = 48 \longrightarrow Z = 7$  (True / False)
- (E)  $2X + 7 = 16 \longrightarrow X = 4$  (True / False)
- (F)  $\frac{D}{6} = 3 \longrightarrow D = 18$  (True / False)





## Unit 5

# Probability

## Vocabulary



- random experiment
- sample space
- event
- probability
- experimental probability
- theoretical probability

## Objectives



**Upon completion of this unit, you will be able to:**

- Identify the random experiment.
- Find the sample space of a random experiment.
- Identify the event of a random experiment.
- Distinguish between experimental and theoretical probabilities.
- Find the probability of a random experiment event.

## (5-1) Random Experiments

A random experiment is one in which it is impossible to predict the outcome with certainty.

**Examples:**

- Throwing a coin and recording the face (heads or tails).
- Rolling a die and recording the number that comes up.
- The result of a football match.

The **sample space** ( $S$ ) of a random experiment is the set of all possible outcomes.

**Examples:**

- For tossing a coin, the sample space is  $S = \{\text{Heads, Tails}\}$ .
- For rolling a die, the sample space is  $S = \{1, 2, 3, 4, 5, 6\}$ .

**1. Find the sample space of each random experiment.**

**A** The result of an examination.

$S = \{\text{Pass, Fail}\}$

**B** Selecting a ball from a box containing balls numbered 1 to 10.

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

**Your Work**

Students' own answers

**Write down your own random experiment's sample space.**



## (5-2) Events and Probability

**An event is a subset of the sample space.**

**Examples:**

- Tossing a coin and it landing on heads: The event is {Heads}.
- Rolling a number greater than 4 on a die: The event is {5, 6}.
- Tossing two coins and landing exactly two tails: The event is {Tails, Tails}.

**Probability is the chance that an event will take place.**

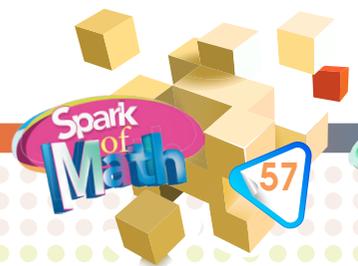
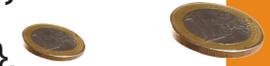
**Examples:**

- When you toss a coin, there's a 50% chance it will land on heads and a 50% chance it will land on tails.
- When you roll a die, there's a 1 in 6 chance of landing on any number from 1 to 6.

## The Probability Scale

**The probability of any event can be shown on a probability scale from 0 to 1:**

- A probability of 0 means the event is impossible to happen (e.g., rolling a "7" on a die).
- A probability of 1 means the event is certain to happen (e.g., rolling a number less than "7" on a die).



## Experimental Probability

Experimental probability is the probability determined by performing an experiment and observing the outcomes.

$$P(\text{Event}) = \frac{\text{Number of times an event occurs}}{\text{Total number of trials}}$$

Example: Imagine tossing a coin 20 times and getting heads 12 times. The experimental probability of getting heads is:

$$P(\text{Head}) = \frac{12}{20} = 0.6 \text{ or } 60\%$$

1. If you roll a die 30 times and get a 4 ten times, what is the experimental probability of rolling a 4?

$$\frac{10}{30} = \frac{1}{3} = 0.33\%$$

## Theoretical Probability

Theoretical probability is the probability determined by calculating the possible outcomes based on the nature of the experiment.

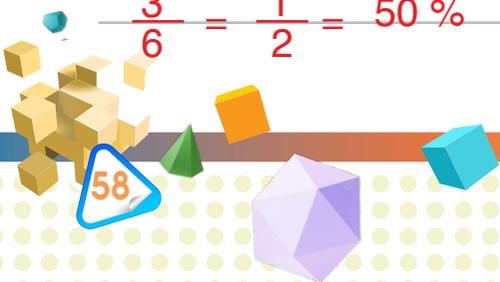
$$P(\text{Event}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

Example: When rolling a six-sided die, the theoretical probability of rolling a 3 is:

$P(3) = \frac{1}{6}$  because there is 1 favorable outcome (rolling a 3) and 6 possible outcomes.

1. What is the theoretical probability of rolling an even number on a six-sided die?

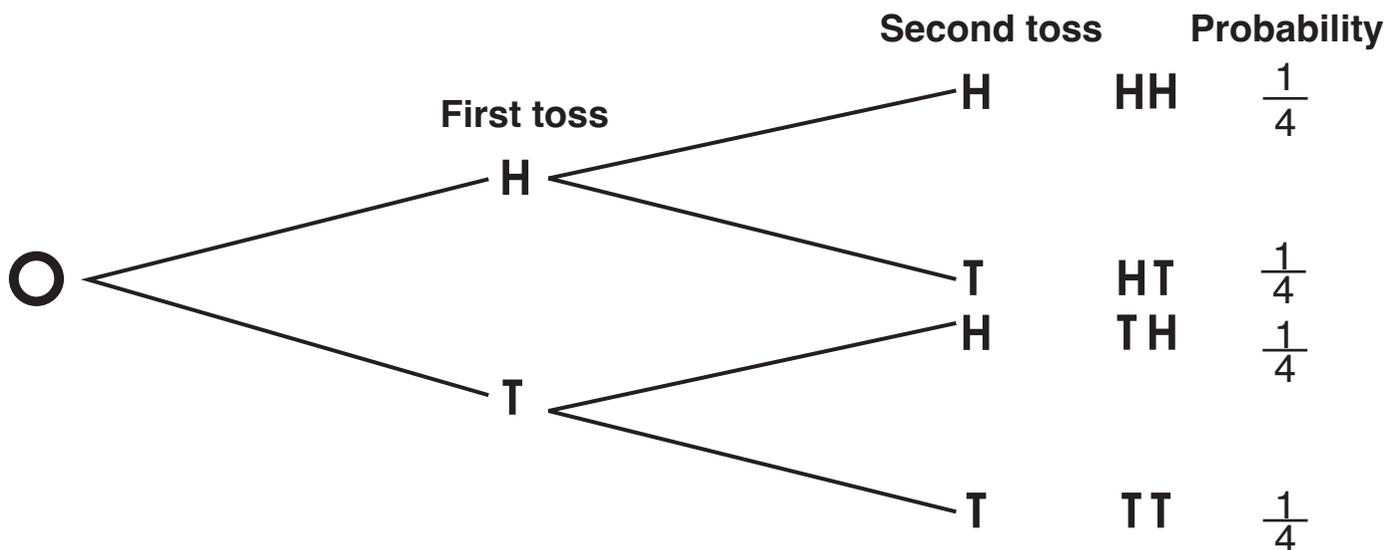
$$\frac{3}{6} = \frac{1}{2} = 50\%$$



## The Probability Tree

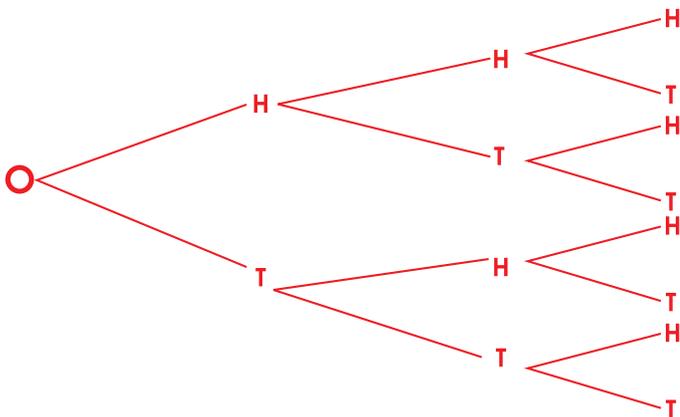
The probability tree helps us determine the number of possible outcomes and analyze the likelihood of each result.

The probability tree below shows the possible outcomes for tossing a coin two times:



The sample space ( $S$ ) = {HH, HT, TH, TT}

1. Draw a tree diagram that shows the results of tossing a coin three times. What is the probability of getting at least two heads?



$(S) = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

The probability of getting at least two heads is  $\frac{4}{8} = \frac{1}{2} = 50\%$ .





## (5-3) Problem Solving

Students' own answers

1. Toss a coin 10 times and record your outcomes in the chart. Then, answer the questions below.

Before you toss, try to make a prediction:

e.g.,

A How many times do you think it will land on heads? 4

B How many times do you think it will land on tails? 6

Toss Number	Heads	Tails	Toss Number	Heads	Tails
1	/		6	/	
2		/	7		/
3	/		8	/	
4	/		9		/
5		/	10	/	

C How many times did it land on heads? 6

Express it as a fraction:  $\frac{6}{10}$

Express it as a percentage: 60%

D How many times did it land on tails? 4

Express it as a fraction:  $\frac{4}{10}$

Express it as a percentage: 40%



**2. In a die-rolling experiment, find the following probabilities and write down the answers as fractions:**

- (A) The probability of getting a 6 is  $\frac{6}{10} = 16.67\%$ .
- (B) The probability of getting an odd number is  $\frac{1}{2} = 50\%$ .
- (C) The probability of getting a number greater than 5 is  $\frac{1}{6} = 16.67\%$ .
- (D) The probability of getting a number divided by 3 is  $\frac{2}{6} = 33.33\%$ .

**3. Roll a die twice and answer the following questions:**

- (A) List all possible outcomes for the two rolls in the sample space.  
 $(S) = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

- (B) Find the probability of getting the sum of "8" when adding the numbers on the two faces.  
The favorable outcomes for obtaining a sum of "8" when rolling a die twice are (2, 6), (3, 5), (4, 4), (5, 3), and (6, 2).

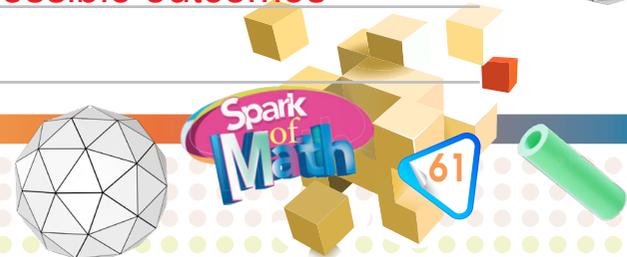
$$P = \text{Number of favorable outcomes} / \text{Total possible outcomes}$$

$$P = \frac{5}{36} \quad P = 13.89\%$$

- (C) Find the probability of getting a sum greater than "8."  
The favorable outcomes for obtaining a sum greater than "8" when rolling a die twice are (3, 6), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 3), (6, 4), (6, 5), and (6, 6).

$$P = \text{Number of favorable outcomes} / \text{Total possible outcomes}$$

$$P = \frac{10}{36} \quad P = 27.78\%$$



4. Roll a die 10 times and record your outcomes in the chart below.

Then, answer the questions that follow. **Students' own answers**

e.g., **Before you start rolling, make a prediction:**

A How many times do you think you will roll a 1? 4

B How many times do you think you will roll a 6? 1

Roll Number	1	2	3	4	5	6
1			/			
2	/					
3						/
4		/				
5				/		
6	/					
7					/	
8						/
9						/
10		/				

C How many times did you roll a 1? 2

Express it as a fraction:  $\frac{2}{10}$

Express it as a percentage: 20%

D How many times did you roll a 6? 3

Express it as a fraction:  $\frac{3}{10}$

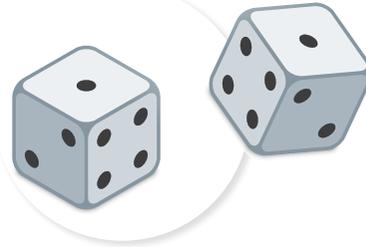
Express it as a percentage: 30%



## Show Your Turn

1. Samer and Fahd tossed two dice 50 times, and note the sum of the rolls of the two dice. Their outcome was the following:

3	5	5	4	6	7	7	5	9	10
12	9	6	5	7	8	7	4	11	6
8	8	10	6	7	4	4	5	7	9
9	7	8	11	6	5	4	7	7	4
3	6	7	7	7	8	6	7	8	9



- A What was their experimental probability of tossing number "5" as a sum?

$$P = \text{Number of times an event occurs} / \text{Total number of trials}$$

$$P = \frac{5}{50} \quad P = 10\%$$

- B What is the theoretical probability of tossing number "5" as a sum?

$$P = \text{Number of favorable outcomes} / \text{Total number of possible}$$

$$\text{outcomes} \quad P = \frac{4}{36} \quad P = 11.11\%$$

2. If you spin a spinner with 5 equally likely sections 40 times and land on the red section 12 times, what is the experimental probability of landing on red?

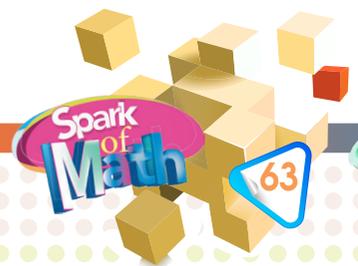
$$P = \text{Number of times an event occurs} / \text{Total number of trials}$$

$$P = \frac{12}{40} \quad P = 30\%$$

3. What is the theoretical probability of rolling a 2 on a six-sided die?

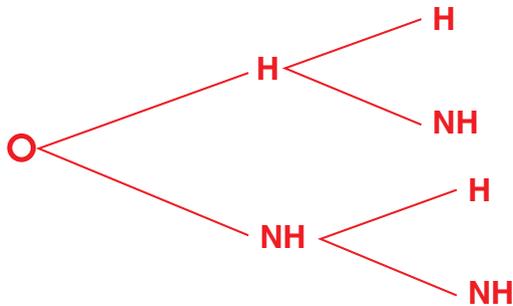
$$P = \text{Number of favorable outcomes} / \text{Total number of possible}$$

$$\text{outcomes} \quad P = \frac{1}{6} \quad P = 16.67\%$$



**4. You have a deck of 52 cards. Each card has a heart on one side and a non-heart on the other side.**

- A** Draw a tree diagram to represent the results of picking two cards in succession from the deck.



- B** What is the total number of possible outcomes when picking two cards in succession from the deck?

$$2 \times 2 = 4$$

- C** What is the probability of selecting a heart side first, and then another heart side?

$$P(HH) = \frac{26}{52} \times \frac{25}{51} = \frac{1}{2} \times \frac{25}{51} = \frac{25}{102} = 24.51\%$$

- D** What is the probability of selecting a non-heart side and then a heart side?

$$P(NH \rightarrow H) = \frac{26}{52} \times \frac{26}{51} = \frac{1}{2} \times \frac{26}{51} = \frac{26}{102} = 25.49\%$$

- E** What is the probability of selecting a heart side first, followed by a non-heart side?

$$P(H \rightarrow NH) = \frac{26}{52} \times \frac{26}{51} = \frac{1}{2} \times \frac{26}{51} = \frac{26}{102} = 25.49\%$$

- F** What is the probability of picking two non-heart sides in succession?

$$P(NH \rightarrow NH) = \frac{26}{52} \times \frac{25}{51} = \frac{1}{2} \times \frac{25}{51} = \frac{25}{102} = 24.51\%$$

